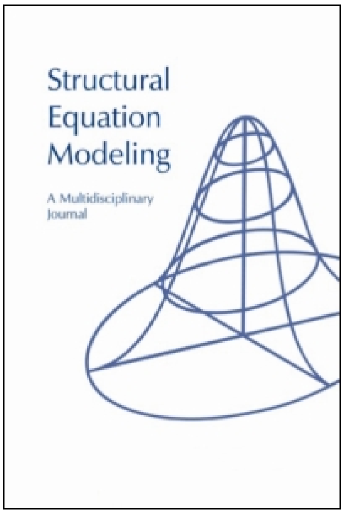

Structural Equation Modeling: A Multidisciplinary Journal
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t775653699

Testing Measurement Invariance in the Target Rotated Multigroup Exploratory Factor Model

Conor V. Dolan [a]; Frans J. Oort [b]; Reinoud D. Stoel [b]; Jelte M. Wicherts [a]
[a] Department of Psychology, University of Amsterdam, [b] Department of Education, University of Amsterdam,

To cite this Article Dolan, Conor V., Oort, Frans J., Stoel, Reinoud D. and Wicherts, Jelte M.'Testing Measurement Invariance in the Target Rotated Multigroup Exploratory Factor Model', Structural Equation Modeling: A Multidisciplinary Journal, 16: 2, 295 — 314

To link to this Article: DOI: 10.1080/10705510902751416

URL: http://dx.doi.org/10.1080/10705510902751416

Structural Equation Modeling, 16:295–314, 2009

ISSN: 1070-5511 print/1532-8007 online
DOI: 10.1080/10705510902751416

Psychology Press
Taylor & Francis Group

Testing Measurement Invariance in the Target Rotated Multigroup Exploratory Factor Model

Conor V. Dolan
Department of Psychology, University of Amsterdam

Frans J. Oort and Reinoud D. Stoel
Department of Education, University of Amsterdam

Jelte M. Wicherts
Department of Psychology, University of Amsterdam

We propose a method to investigate measurement invariance in the multigroup exploratory factor model, subject to target rotation. We consider both oblique and orthogonal target rotation. This method has clear advantages over other approaches, such as the use of congruence measures. We demonstrate that the model can be implemented readily in the freely available Mx program. We present the results of 2 illustrative analyses, one based on artificial data, and the other on real data relating to personality in male and female psychology students.

Most work in the area of multigroup factor analysis is based on the confirmatory factor model; that is, a model in which rotational indeterminacy is resolved by the specification of a sufficient number of fixed zero-factor loadings (Jöreskog, 1971; Little, 1997; Millsap & Everson, 1991; Sörbom, 1974). Zero-factor loadings are often based on substantive considerations, such as the design of the measurement instrument, and can thus be well motivated. However, there are situations in which an exploratory approach might be more appropriate. First, as pointed out by Browne (2001; see also Ferrando & Lorenzo-Seva, 2000), the initially confirmatory model is often modified extensively on the basis of modification indexes and other results, rather than on the basis of a preconceived, theory-driven strategy. Second, a confirmatory approach is inappropriate when fixed zero loadings are judged to be too restrictive on theoretical grounds. Notably McCrae, Zonderman, Costa, Bond, and Paunonen (1996) argued that zero

Correspondence should be addressed to Conor V. Dolan, Department of Psychology, University of Amsterdam, Roetersstraat 15, 1011HZ Amsterdam, Netherlands. E-mail: c.v.dolan@uva.nl

loadings are unsuited in factor analyses of the NEO Personality Inventory-Revised (NEO-PI-R), a psychometric instrument designed to measure the Big Five personality dimensions (Costa & McCrae, 1992). In the context of group comparisons, specifically, they recommended the combined use of exploratory FA, target rotation, and congruence measures to evaluate the similarity of factor loadings over groups. Similarly, Hattie and Fraser (1988) argued that zero loadings are unsuited in the FA of a psychometric test designed to measure the dimensions of information processing as proposed by Luria (1978). Hattie and Fraser (1988), who were not concerned with group comparisons, proposed the application of parameter bounds to include small, but nonzero loadings.

The aim of this article is to present a multigroup exploratory factor model subject to target rotation, in which we can investigate measurement invariance. Rather than using congruence measures, we propose fitting the model using a freely available structural equation modeling (SEM) program. We assume that one has at their disposal a target rotation matrix, which might have been obtained in some other FA, or can be specified de novo. In the latter case, one has a definite idea about which indicators load on common factors, but one is unwilling to specify fixed zero loading.

The work that has been done in the area of multigroup exploratory factor analysis (MG-EFA), although relatively minimal, provides the basis for the approach used here. McArdle and Cattell (1994) provided a clear and detailed account of minimal constraints required to achieve model identification in the multigroup setting. Based on this work, Hessen, Dolan, and Wicherts (2006) investigated (a) the establishment of measurement invariance independent of a confirmatory structure, (b) the relative loss in power to detect violations of measurement invariance in the exploratory model, and (c) the use of the exploratory structure as a baseline model for an evaluation of measurement invariance in a confirmatory model. They presented an MG-EFA, in which rotational indeterminacy was solved by constraining the in-product of the factor loading matrix $\boldsymbol{\Lambda}$ (i.e., $\boldsymbol{\Lambda}'\boldsymbol{\Lambda}$) to be diagonal. This constraint has certain advantages over the practice of introducing arbitrary fixed zero-factor loadings (see McArdle & Cattell, 1994; Millsap, 2001). Although the constraint mentioned removes rotational indeterminacy, it generally does not result in an interpretable solution. Subsequent rotation was not considered by Hessen et al. (2006).

In what follows, we present the MG-EFA models, subject to target rotation, along with the requirements associated with measurement invariance, as defined by Mellenbergh (1989), and discussed in detail in the context of the common factor model by Meredith (1993). We consider both oblique and orthogonal target rotation. We present illustrative analyses of simulated and real data relating to personality. We conclude with a discussion.

MULTIGROUP EXPLORATORY FACTOR ANALYSIS

Let $\boldsymbol{y}_{ij}$ denote a p-dimensional random vector of indicators observed in subject j $(j = 1 \ldots N_i)$ in population i. We consider just two populations ($i = 1, 2$), but the extension to three or more groups is straightforward. The exploratory factor model for the observations is (e.g., Lawley & Maxwell, 1971):

$$\boldsymbol{y}_{ij} = \boldsymbol{\tau}_i + \boldsymbol{\Lambda}_i \boldsymbol{\eta}_{ij} + \boldsymbol{\varepsilon}_{ij}, \quad (1)$$

where $\boldsymbol{\varepsilon}$ is the p-dimensional random vector of mutually uncorrelated residuals, $\boldsymbol{\eta}_{ij}$ is a q-dimensional vector of common factor scores, $\boldsymbol{\tau}_i$ is a p-dimensional vector of regression intercepts, and $\boldsymbol{\Lambda}_i$ is the $p \times q$ matrix of factor loadings. We assume throughout that $q > 1$, as $q = 1$ implies a confirmatory model. Assuming $\boldsymbol{\eta}$ and $\boldsymbol{\varepsilon}$ are uncorrelated, the implied means and covariance structure is:

$$\boldsymbol{\Sigma}_i = \boldsymbol{\Lambda}_i \boldsymbol{\Psi}_i \boldsymbol{\Lambda}_i' + \boldsymbol{\Theta}_i, \text{ and } \boldsymbol{\mu}_i = \boldsymbol{\tau}_i + \boldsymbol{\Lambda}_i \boldsymbol{\alpha}_i,$$

where the $\boldsymbol{\Psi}_i$ is the $q \times q$ covariance matrix of the common factors, and $\boldsymbol{\Theta}_i$ is the diagonal covariance matrix of the residuals $\boldsymbol{\varepsilon}$. The p-dimensional vector $\boldsymbol{\tau}_i$ contains the intercepts, and the q-dimensional vector $\boldsymbol{\alpha}_i$ contains the factor means.

Scaling Constraints

As it stands, this model is not identified. To achieve identification, we have to impose two types of constraints, *rotational constraints* and *scaling constraints.* The scaling constraints serve to impose a scale on the latent variables $\boldsymbol{\eta}$ (Bollen, 1989). Usually this is achieved by fixing the means and variances of the latent variables $\boldsymbol{\eta}$ to

- Sensible fixed values (e.g., 0 and 1, respectively).
- Values that depend directly on the scale of observed indicator (e.g., by fixing certain values in $\boldsymbol{\Lambda}_i$ to 1).

However, we refer the reader to Little, Slegers, and Card (2006) for a third method of scaling, and for a discussion of the relative merits of the various methods of scaling in the confirmatory context. With respect to the latent means, we initially fixed the factor means to equal zero ($\boldsymbol{\alpha}_i = 0$) and estimated the intercepts freely ($\boldsymbol{\mu}_i = \boldsymbol{\tau}_i$). As we see later, this constraint can be relaxed in certain multigroup models (Sörbom, 1974).

Rotational Constraints

The rotational constraints serve to remove rotational indeterminacy by fixing a minimal number of parameters. There are several ways to achieve this (McArdle & Cattell, 1994):

- Traditionally in EFA, the constraint is imposed that the off-diagonals of the matrix $\boldsymbol{\Lambda}_i'\boldsymbol{\Lambda}_i$ be zero, Off-diag$(\boldsymbol{\Lambda}_i'\boldsymbol{\Lambda}_i) = \mathbf{0}$, in combination with uncorrelated common factors, Off-diag$(\boldsymbol{\Psi}_i) = 0$ (Lawley & Maxwell, 1971).[1] In the single group case, this is usually combined with scaling is $\boldsymbol{\Psi}_i$, so that $\boldsymbol{\Psi}_i = \mathbf{I}$.

[1]Actually Lawley and Maxwell (1971) discussed the constraint Off-diag$(\boldsymbol{\Lambda}'\boldsymbol{\Theta}^{-1}\boldsymbol{\Lambda}) = \mathbf{0}$. For purposes of identification, Off-diag$(\boldsymbol{\Lambda}'\boldsymbol{\Lambda}) = 0$ serves just as well (Jöreskog, 1978).

- Another possibility, known as echelon rotation (McDonald, 1999), is to fix the $q*(q-1)/2$ factor loadings to zero, while retaining $\mathbf{\Psi}_i = \mathbf{I}$. For instance, given $q = 3$:

$$\mathbf{\Lambda}_i = \begin{matrix} \boldsymbol{\lambda}_{11} & 0 & 0 \\ \boldsymbol{\lambda}_{21} & \boldsymbol{\lambda}_{22} & 0 \\ \boldsymbol{\lambda}_{31} & \boldsymbol{\lambda}_{32} & \boldsymbol{\lambda}_{33} \\ \boldsymbol{\lambda}_{41} & \boldsymbol{\lambda}_{42} & \boldsymbol{\lambda}_{43} \\ \dots & \dots & \dots \\ \boldsymbol{\lambda}_{p1} & \boldsymbol{\lambda}_{p2} & \boldsymbol{\lambda}_{p3} \end{matrix}. \tag{2}$$

- Reference variable rotation (e.g., Ferrando & Lorenzo-Seva, 2000; Jöreskog, 1978) is yet another way to arrive at an identified model by fixing all but one factor loading to zero in q distinct columns of $\mathbf{\Lambda}_i$, while allowing the common factor to correlate. For instance:

$$\mathbf{\Lambda}_i = \begin{matrix} \boldsymbol{\lambda}_{11} & 0 & 0 \\ 0 & \boldsymbol{\lambda}_{22} & 0 \\ 0 & 0 & \boldsymbol{\lambda}_{33} \\ \boldsymbol{\lambda}_{41} & \boldsymbol{\lambda}_{42} & \boldsymbol{\lambda}_{43} \\ \dots & \dots & \dots \\ \boldsymbol{\lambda}_{p1} & \boldsymbol{\lambda}_{p2} & \boldsymbol{\lambda}_{p3} \end{matrix}. \tag{3}$$

  The exact position of the zero loadings in the reference variable rotation or the echelon rotation is not completely arbitrary. As discussed by Millsap (2001), an infelicitous choice could result in computational problems (see also McArdle & Cattell, 1994). Generally, this can be avoided by trial and error, or by using information obtained in a standard EFA (Jöreskog, 1978).

With the minimal identifying constraints in place, we have an identified model, which we denote by the baseline MG-EFA model:

$$\mathbf{\Sigma}_i = \mathbf{M}_i\mathbf{M}_i' + \mathbf{\Theta}_i, \text{ and } \boldsymbol{\mu}_i = \boldsymbol{\tau}_i, \tag{4}$$

where $\mathbf{M}_i = \mathbf{\Lambda}_i\mathbf{\Psi}_i^{1/2}$ (depending on the exact rotational and scaling constraints, $\mathbf{\Psi}_i$ might equal $\mathbf{I}$). This solution can be rotated in each group independently. Given known rotation matrices $\mathbf{R}_i$, we have with $\mathbf{B}_i = \mathbf{M}_i\mathbf{R}_i$,

$$\mathbf{\Sigma}_i = \mathbf{B}_i\mathbf{\Phi}_i\mathbf{B}_i' + \mathbf{\Theta}_i, \text{ and } \boldsymbol{\mu}_i = \boldsymbol{\tau}_i. \tag{5}$$

The covariance matrices $\mathbf{\Phi}_i$ equal $[\mathbf{R}_i^{-1}\mathbf{R}_i^{-1'}]$. If $\mathbf{R}_i$ is an orthogonal rotation matrix, $\mathbf{R}_i^{-1}$ equals $\mathbf{R}_i'$, so that $\mathbf{R}_i\mathbf{R}_i' = \mathbf{I}$. As required, the rotation does not affect goodness of fit (i.e., $\mathbf{\Sigma}_i$ in Equation 5 equals $\mathbf{\Sigma}_i$ in Equation 4). With this model in place we consider the constraints necessary for measurement invariance, and, subsequently the actual determination of $\mathbf{R}_i$, in target rotation.

Measurement Invariance

We briefly consider the successive constraints, implied by measurement invariance. The constraints, which are derived by Meredith (1993; see also Chen, Sousa, & West, 2005; Horn & McArdle, 1992; Little, 1997; Millsap & Kwok, 2004), concern the equality over groups of factor loadings, intercepts, and residual variances. The constraints are based on the application of Mellenbergh's definition of unbiasedness to the common factor model (Meredith, 1993; see Dolan, Roorda, & Wicherts, 2004, for a discussion in conceptual terms). It is useful to consider the constraints successively, because each progressively constrained model is nested under less constrained models. This provides us with the possibility of likelihood ratio testing to evaluate each constraint in succession (e.g., Dolan, 2000; van der Sluis et al., 2005; Wicherts et al., 2004), and so to facilitate the identification of model violations, if any. We present the constraints in three steps.

The first step involves the equality over groups of the factor loadings. With the first group as the reference group by setting $\mathbf{M} = \mathbf{\Lambda}_1\mathbf{\Psi}_1^{1/2}$, the imposition of this equality constraint results in the following model for the covariance structure

$$\mathbf{\Sigma}_1 = \mathbf{M}\mathbf{M}^t + \mathbf{\Theta}_1, \mathbf{\Sigma}_2 = \mathbf{M}\mathbf{\Psi}_2\mathbf{M}^t + \mathbf{\Theta}_2, \text{ and } \boldsymbol{\mu}_i = \boldsymbol{\tau}_i. \quad (6)$$

The choice of the reference group is not arbitrary, as we discuss later. The presence of the positive definite matrix $\mathbf{\Psi}_2$ is required, as the covariance matrix of the unrotated common factors in Group 2 are not necessarily equal to that in Group 1. Hypotheses concerning $\mathbf{\Psi}_2$ (e.g., $\mathbf{\Psi}_2$ is diagonal) are amenable to empirical testing. This model is sometimes referred to as the metric invariance model (Horn & McArdle, 1992; Millsap & Kwok, 2004). As $\mathbf{M}$ is equal over groups, the rotation matrix $\mathbf{R}$ is also equal. So given rotation, we have $\mathbf{B} = \mathbf{MR}$, and

$$\mathbf{\Sigma}_i = \mathbf{B}\mathbf{\Phi}_i\mathbf{B}' + \mathbf{\Theta}_i,$$

where $\mathbf{\Phi}_1 = [\mathbf{R}^{-1}\mathbf{R}^{-1'}]$ and $\mathbf{\Phi}_2 = [\mathbf{R}^{-1}\mathbf{\Psi}_2\mathbf{R}^{-1'}]$. If $\mathbf{R}$ is an orthogonal rotation matrix, $\mathbf{\Phi}_1 = \mathbf{I}$. However, $\mathbf{\Phi}_2$ need not be diagonal. This is the case only if $\mathbf{\Psi}_2 = \mathbf{I} * \gamma$, where γ is a positive scalar ($\gamma = 1$, possibly). This can be tested by constraining $\mathbf{\Psi}_2$ to equal $\mathbf{I}*\gamma$, or by constraining $\mathbf{\Phi}_2$ to be diagonal.

So far the mean structure was saturated ($\boldsymbol{\mu}_i = \boldsymbol{\tau}_i$). The second step involves the equality of the intercepts $\boldsymbol{\tau}_1 = \boldsymbol{\tau}_2$ (i.e., the strong factorial invariance model). With the appropriate constraint in place, the factor means can be included, resulting in Equation 6 in combination with

$$\boldsymbol{\mu}_i = \boldsymbol{\tau} + \mathbf{M}\boldsymbol{\alpha}_i = \boldsymbol{\tau} + \mathbf{MR}\boldsymbol{\kappa}_i = \boldsymbol{\tau} + \mathbf{B}\boldsymbol{\kappa}_i, \quad (7)$$

where $\boldsymbol{\kappa}_i = \mathbf{R}^{-1}\boldsymbol{\alpha}_i$. Although the factor means are not identified, the differences in factor means are (see Sörbom, 1974). The factor means are, therefore, constrained to zero in Group 1 (say), and the factor mean differences are estimated in Group 2:

$$\boldsymbol{\mu}_1 = \boldsymbol{\tau}, \text{ and } \boldsymbol{\mu}_2 = \boldsymbol{\tau} + \mathbf{M}\boldsymbol{\delta}_{\boldsymbol{\alpha}} = \boldsymbol{\tau} + \mathbf{MR}\boldsymbol{\delta}_{\boldsymbol{\kappa}} = \boldsymbol{\tau} + \mathbf{B}\boldsymbol{\delta}_{\boldsymbol{\kappa}}, \quad (8)$$

where $\boldsymbol{\delta}_{\boldsymbol{\alpha}} = (\boldsymbol{\alpha}_2 - \boldsymbol{\alpha}_1)$ and $\boldsymbol{\delta}_{\boldsymbol{\kappa}} = (\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1)$. This model is referred to as the strong factorial invariance model (Meredith, 1993). Little, Slegers, and Card (2006) presented alternative methods of parameterizing this model. One method is to fix the intercept of a single indicator of each common factor to zero, and freely estimate the latent means. Another method is to impose so-called effects-coding constraints, in which the intercepts of the indicators of each factor sum to zero. These methods would seem to be more suitable in the confirmatory context where indicators of a common factor can be identified on the basis of prior information.

In the third step the covariance matrices of the residuals are constrained to be equal:

$$\mathbf{\Sigma}_i = \mathbf{B\Phi}_i\mathbf{B}' + \mathbf{\Theta}. \tag{9}$$

Equation 9 in combination with Equation 8c represents the strict factorial invariance model; that is, measurement invariance in the factor model. This model satisfies Mellenbergh's (1989) definition of unbiasedness in the common factor model.[2]

Rotation

The rotation matrices $\mathbf{R}_i$ can be obtained by optimizing a suitable rotation criterion. We refer the reader to Browne (2001) and Bernaards and Jennrich (2005) for a discussion of possible rotation criteria. Here we limit our attention to oblique and orthogonal target rotations. An oblique target rotation matrix is obtained by minimizing the least squares criterion:

$$Q_T(\mathbf{M}_i\mathbf{R}_i) = \text{trace}\{(\mathbf{M}_i\mathbf{R}_i - \mathbf{T}_i)'(\mathbf{M}_i\mathbf{R}_i - \mathbf{T}_i)\}, \tag{10}$$

where $\mathbf{T}_i$ is a target matrix, which can be chosen to reflect a theoretical factor structure or can be a factor loading matrix obtained in prior analysis (see McCrae et al., 1996). In the former case, $\mathbf{T}_i$ usually contains ones (in the position of the expected nonzero factor loadings) and zeros. However, this rotation criterion does not exclude nonzero loadings in the positions of $\mathbf{M}_i\mathbf{R}_i$ associated with zeros in $\mathbf{T}_i$. The closed form solution of Equation 10 is

$$\mathbf{R}_i = (\mathbf{M}_i'\mathbf{M}_i)^{-1}\mathbf{M}_i'\mathbf{T}_i \tag{11}$$

(e.g., Basilevsky, 1983). Once $\mathbf{R}_i$ has been obtained, it is normalized, such that diag($\mathbf{R}_i\mathbf{R}_i'$) $=$ diag($\mathbf{I}$). Schönemann (1966) presented a solution for $\mathbf{R}_i$ in Equation 10, subject to the constraint $\mathbf{R}_i\mathbf{R}_i' = \mathbf{I}$; that is, orthogonality of the rotation. Let $\mathbf{W}_i\mathbf{D}_i\mathbf{W}_i'$ denote the eigenvalue decomposition of the symmetric $q \times q$ matrix $\mathbf{\Lambda}_i'\mathbf{T}_i\mathbf{T}_i'\mathbf{\Lambda}_i$, and $\mathbf{V}_i\mathbf{D}_i\mathbf{V}_i'$ denote the eigenvalue decomposition of symmetric $q \times q$ matrix $\mathbf{T}_i'\mathbf{\Lambda}_i\mathbf{\Lambda}_i'\mathbf{T}_i$. That is, the diagonal $q \times q$ matrix $\mathbf{D}_i$ contains the q positive eigenvalues, and the $q \times q$ matrices $\mathbf{W}_i$ and $\mathbf{V}_i'$ contain the orthogonal eigenvectors ($\mathbf{W}_i\mathbf{W}_i' = \mathbf{V}_i\mathbf{V}_i' = \mathbf{I}$). Schönemann (1966) demonstrated that $\mathbf{R}_i = \mathbf{WV}'$. Alternatively one can carry out a singular value decomposition of $\mathbf{\Lambda}_i'\mathbf{T}_i$; that is, $\mathbf{\Lambda}_i'\mathbf{T}_i = \mathbf{WD}_i^{1/2}\mathbf{V}'$. As noted by Schönemann (1966), care should be taken that the orientation of the

[2]Sometimes additional equality constraints are placed on $\mathbf{\Psi}_i$ or $\mathbf{\Phi}_i$. However we do not consider these, as they do not follow from Mellenbergh's (1989) definition of measurement invariance. In the presence of latent mean differences, the equality of $\mathbf{\Psi}_i$ or $\mathbf{\Phi}_i$ would seem to require a theoretical account of how mean differences in the common factors can arise in the absence of differences in the covariance matrices of the common factors. We note that the assumption of homoskedasticity in multivariate analysis of variance (i.e., equality of covariance matrices) represents a statistical, not a substantive, constraint.

eigenvectors of $\mathbf{W}$ and $\mathbf{V}$ be chosen such that the diagonal elements of $\mathbf{D}_i^{1/2}$ be positive. Later, we achieve this by premultiplying the matrix $\mathbf{W}$ by the matrix $\mathbf{E} = \mathbf{D}_i^{1/2}\mathbf{D}_i^{*-1/2}$, where $\mathbf{D}_i^{*1/2}$ is the absolute value of $\mathbf{D}_i^{1/2}$. The matrix $\mathbf{E}$ is a diagonal matrix containing ones, negative ones, or both on the diagonal. The matrix $\mathbf{D}_i^{1/2}$ can be calculated as $\mathbf{W}'\mathbf{D}_i^{1/2}\mathbf{V}$. So we have

$$\mathbf{R}_i = \mathbf{EWV}' \tag{12}$$

STRATEGY

We assume the data to be independently and normally distributed $\mathbf{y}_{ij} \sim N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$, so that the parameters can be estimated by minimizing the normal theory maximum likelihood ratio function. Furthermore we assume that the sample of independent cases is sufficiently large to justify the application of the likelihood ratio testing of specific hypotheses (Bollen, 1989; Lawley & Maxwell, 1971).

If one is interested only in establishing measurement invariance, and not concerned with the actual interpretation of the results, one can simply fit the models, given any minimal identifying constraints. This was the situation discussed in Hessen et al. (2006). In addition, if the factor loadings in the unrotated model are found to be unequal over groups, there might be little point in continuing with a multigroup approach. One then might resort to single-group analyses using, for instance, the CEFA program (Browne, Cudeck, Tateneni, & Mels, 1998), which includes a wide variety of rotation criteria, and provides standard errors of rotated factor loadings (see Cudeck & O'Dell, 1994). We therefore first test whether the strong or strict factorial invariance model fits adequately, as outlined in Hessen et al. (2006). Subsequently, we fit the model subject to the orthogonal or oblique target rotation. The proposed strategy involves the following steps:

1. Establish strong or strict factorial invariance in an unrotated MG-EFA (Hessen et al., 2006) The model can be estimated and tested with any multigroup SEM computer program.
2. Choose an orthogonal or oblique target $\mathbf{T}$.
3. Refit the strong or strict factorial invariance model subject to target rotation given $\mathbf{T}$, and calculate confidence intervals of the parameters of interest.

We use the Mx program (Neale, Boker, Xie, & Maes, 2003) to fit the models, because it allows one full freedom with respect to model specification. That is, using Mx we can fit the exploratory factor model subject to target rotation in a single analysis. The closed form solutions of the orthogonal and oblique target rotation can be specified directly in Mx (Equations 11 and 12). In addition, Mx can calculate confidence intervals of any parameter estimate, be it freely estimated, or transformed, as is the case with the parameters in the matrix $\mathbf{B}$, the matrices $\boldsymbol{\Phi}_i$, or the differences in factor means $\boldsymbol{\delta}_{\boldsymbol{\kappa}}$. The Mx confidence intervals are based on the likelihood profile (Neale & Miller, 1997). These are often more accurate than standard errors based on the information matrix. We do note that in large models the calculation of confidence intervals can take a long time.

The Choice of Reference Group

In testing strong or strict measurement invariance in MG-EFA, the parameterization and the choice of the reference group can affect the exact results, following rotation. To see this, consider the model in which Group 1 is the reference group. Given strict factorial invariance, we have

$$\mathbf{\Sigma}_1 = \mathbf{MM}' + \mathbf{\Theta} = \mathbf{B}[\mathbf{R}^{-1}\mathbf{R}^{-1'}]\mathbf{B}' + \mathbf{\Theta}, \tag{13a}$$

$$\mathbf{\Sigma}_2 = \mathbf{M\Psi}_2\mathbf{M}' + \mathbf{\Theta} = \mathbf{B}[\mathbf{R}^{-1}\mathbf{\Psi}_2\mathbf{R}^{-1'}]\mathbf{B}' + \mathbf{\Theta}. \tag{13b}$$

The dependency on the results arises because $\mathbf{M}$, as defined in Group 1, usually includes information that is particular to this group. This is most evident if $\mathbf{M}$ is calculated using the reference variable formulation: $\mathbf{M} = \mathbf{\Lambda\Psi}_1^{1/2}$, where the structure of $\mathbf{\Lambda}$ is conveyed in Equation 4, and $\mathbf{\Psi}_1$ is the positive definite covariance matrix in Group 1. Had we chosen Group 2 as the reference group, we would have

$$\mathbf{\Sigma}_1 = \mathbf{M\Psi}_1\mathbf{M}' + \mathbf{\Theta} = \mathbf{B}[\mathbf{R}^{-1}\mathbf{\Psi}_1\mathbf{R}^{-1'}]\mathbf{B}' + \mathbf{\Theta}, \tag{14a}$$

$$\mathbf{\Sigma}_2 = \mathbf{MM}' + \mathbf{\Theta} = \mathbf{B}[\mathbf{R}^{-1}\mathbf{R}^{-1'}]\mathbf{B}' + \mathbf{\Theta}, \tag{14b}$$

so that now $\mathbf{M}$ constrains information specific to Group 2. The consequence of this is that the exact results in terms of the parameter estimates vary depending on the choice of reference group. This is not the case, however, if $\mathbf{\Psi}_2$ in Equation 13b or $\mathbf{\Psi}_1$ in Equation 14a happens to equal $\mathbf{I} * \gamma$, where γ is a positive scalar. As mentioned earlier, this is amenable to statistical testing.

Next, we present the results of two illustrative analyses. The first concerns the analyses of exact population matrices. The second concerns the analysis of the NEO-PI-R in male and female psychology students. The analyses were carried out using LISREL (Jöreskog & Sörbom, 1999) and Mx (Neale et al., 2003).

ILLUSTRATION 1: ARTIFICIAL DATA

In the first illustration, we analyzed exact population matrices to demonstrate how the choice of reference group can affect the exact parameter estimates. Two population matrices were created using the parameter values shown in Table 1.

We fit the models to both oblique and orthogonal target rotations, using the following target matrix:

$$\mathbf{T} = \begin{matrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{matrix}$$

The Mx scripts are given in the Appendix (these include the summary statistics). Table 2 contains the estimated parameters with both Group 1 and Group 2 specified as the reference

TABLE 1
Parameter Values Used to Create Artificial Data

$\mathbf{\Lambda}$ = .80	.00	$\mathbf{\Psi}_1$ = 1			$\boldsymbol{\alpha}_1$ = [0 0]
.75	.15		.35	1	
.70	.20	$\mathbf{\Psi}_2$ =	.9		$\boldsymbol{\alpha}_2$ = [0.5 −.5]
.20	.70		.4	1.1	
.10	.80				
.00	.75				
diag($\mathbf{\Theta}$) = [.5 .5 .5 .5 .5 .5]					
$\boldsymbol{\tau}$ = [5 5 5 5 5 5]					

Note. The associated summary statistics are shown in the Appendix. The results of the analysis are shown in Table 2.

TABLE 2
Results of Fitting the Multigroup Exploratory Factor Analysis to Artificial Data Subject to Orthogonal and Oblique Target Rotation

	Group 1 Reference		*Group 2 Reference*	
Raw estimates oblique target rotation				
$\mathbf{B}$ =	0.861	−.121	0.828	−.127
	0.788	0.046	0.757	0.048
	0.727	0.107	0.699	0.112
	0.125	0.716	0.120	0.751
	0.004	0.837	0.004	0.878
	−.097	0.799	−.093	0.838
$\mathbf{\Phi}$ =	1.000		1.081	
	0.557	1.000	0.552	0.908
$\mathbf{\Phi}$ =	0.925		1.0	
	0.605	1.100	0.599	1.0
$\boldsymbol{\alpha}_1$ =	0.0	0.0	−.421	0.400
$\boldsymbol{\alpha}_2$ =	0.405	−.420	0.0	0.0
Raw estimates orthogonal target rotation				
$\mathbf{B}$ =	0.788	0.133	0.744	0.150
	0.767	0.272	0.731	0.294
	0.727	0.313	0.695	0.336
	0.329	0.720	0.341	0.754
	0.249	0.802	0.271	0.838
	0.141	0.736	0.167	0.768
$\mathbf{\Phi}_1$ =	1.0		1.145	
	0.0	1.0	−.066	0.927
$\mathbf{\Phi}_2$ =	0.874		1.0	
	0.059	1.085	0.0	1.0
$\boldsymbol{\alpha}_1$ =	0.0	0.0	−.665	0.633
$\boldsymbol{\alpha}_2$ =	0.613	−.627	0.0	0.0

Note. See Appendix for Mx scripts.

group. In both cases we used echelon rotation (fixing element in position [1,2] in $\mathbf{\Lambda}$ to zero), and scaling in $\mathbf{\Psi}_1$ or $\mathbf{\Psi}_2$, depending on the choice of reference group.

The factor loadings and latent mean differences are shown in Table 2. The $\chi^2(26)$ for the model is zero, as expected. There are slight differences associated with the choice of reference group. As explained earlier, these have their origins in the difference in the matrices $\mathbf{\Psi}_1$ and $\mathbf{\Psi}_2$. The χ^2 test of no latent mean differences, which depends neither on the choice of reference group, nor on the choice of rotation criterion, is $\chi^2(2) = 58.4$.

ILLUSTRATION 2: NEO-PI-R

The second illustration concerns the NEO-PI-R, a psychometric test designed to measures the Big Five personality dimensions (Costa & McCrae, 1992). These are Neuroticism (N), Extraversion (E), Openness to experience (O), Agreeableness (A), and Conscientiousness (C). McCrae et al. (1996) rejected MG-CFA, because they judged it to be too restrictive. They proposed the use of target rotation and congruence measures to establish the equality of factor loadings over groups. Congruence measures might well suffice to establish metric invariance in the EFA, and bootstrap procedures have been proposed to actually test such measures (Chan, Ho, Leung, Chan, & Yung, 1999; McCrae et al., 1996). However, equality of factor loadings is a necessary, not a sufficient, condition for measurement invariance. The interpretation of mean differences in terms of the Big Five personality dimensions (e.g., McCrae & Costa, 1997) first requires one to establish strong or strict factorial invariance (i.e., a model including structured means).

We illustrate this using the 30 NEO-PI-R facet scores obtained in a sample of 139 male and 361 female psychology students at the University of Amsterdam. To investigate measurement invariance with respect to sex, we first fitted an unconstrained five-factor MG-CFA using echelon rotation (Equation 3b). We used LISREL to this end, although Mx can also be used. To evaluate the goodness of fit, we considered the chi-square, the root mean squared error of approximation (RMSEA), the expected cross-validation index (ECVI; this conveys the same information as Akaike's Information Criterion; see Browne & Cudeck, 1993), the comparative fit index (CFI), and the nonnormed fit index (NNFI). Bollen and Long (1993) recommended the use of several, diverse indexes in evaluating goodness of fit. For a discussion of these and other indexes we refer the reader to Browne and Cudeck (1993), Jöreskog (1993), and Schermelleh-Engel, Moosbrugger, and Müller (2003). The indexes of the models are shown in Table 3.

The goodness of fit of the unconstrained model is $\chi^2(590) = 1{,}408$. In this Model 1, the only source of misfit is the diagonality of the covariance matrices of the residuals (assuming the number of factors is correctly specified). LISREL identified four large off-diagonals in the female sample[3] and one large off-diagonal in the male sample.[4] Freeing these resulted in Model 2 with $\chi^2(585) = 1{,}262$. Judging by the drop in χ^2 of 146 ($df = 5$), and the changes in RMSEA, ECVI, NNFI, and CFI, the modification is justified (see Table 3).

[3]The correlated residual concerns the following facet pairs: self-discipline (C29) and ambition (C28), reliability (C3) and sincerity (A2), energy (E4) and changes (O4), and modesty (A5) and ideas (O5).

[4]The correlated residual concerns the facet pair sociability (E2) and adventurism (E5).

TABLE 3
Goodness-of-Fit Criteria for Unrotated MG-EFA of Revised NEO Personality Inventory Data in 139 Male and 361 Female Psychology Students at the University of Amsterdam

Model	*df*	χ^2	*RMSEA 90% CIs*	*ECVI*	*NNFI*	*CFI*
1 MG-EFA	590	1,408	.079 (.074, .084)	4.64	.889	.925
2 Rev. MG-EFA[a]	585	1,262	.070 (.065, .071)	4.26	.907	.938
3 Metric inv.	710	1,486	.066 (.062, .071)	4.12	.913	.929
4 Strong fact. inv.	735	1,574	.068 (.063, .073)	4.22	.909	.923
5 Rev. strong fact inv.[b]	733	1,526	.066 (.061, .071)	4.11	.914	.927
Comparison	*df*	χ^2	*p*			
1 versus 2	5	146	<.01			
2 versus 3	125	224	<.01			
4 versus 3	25	88	<.01			
5 versus 4	2	48	<.01			
5 versus 3	23	40	.015			

Note. MG-EFA = multigroup exploratory factor analysis; RMSEA = root mean squared error of approximation; CIs = confidence intervals; ECVI = expected cross-validation index; NNFI = Nonnormed Fit Index; CFI = comparative fit index.

[a]Revised by adding five covariances among residuals (four in the female sample and one in the male sample).

[b]Revised by removing the equality constraints on the intercepts of two facets (A2 and A5).

Constraining the factor loadings to be equal over sex produced Model 3 with $\chi^2(710) = 1{,}486$. The comparison with Model 2 revealed an increase in χ^2 of 224 with 125 *df.* However, the other fit indexes favor this model. In addition, the modification indexes of the constrained factor loadings were quite acceptable, as illustrated in Figure 1.

Accepting this model, we fitted the strong factorial invariance model. This resulted in Model 4 with $\chi^2(735) = 1{,}574$. The increase in χ^2 is 88 with 25 *df.* LISREL indicated two large modification indexes of the intercepts associated with two Agreeableness facets (A2 sincerity and A5 modesty). We relaxed the equality constraints on the intercepts of these facets. The $\chi^2(733)$ of the resulting Model 5 equals 1,526 (a decrease in χ^2 of 48 for 2 *df*). The difference in χ^2 between Model 3 (metric invariance) and Model 5 (partial strong factorial invariance) is 40 with 23 *df* ($p = .015$). In view of the difference in the covariance of the males (one free off-diagonal) and females (four free off-diagonals), we did not consider the partial strict factorial invariance model. For present purposes, we accept Model 5. The ECVI favors this model (4.11) and the RMSEA is acceptable (.066), as are the NNFI and CFI (.914 and .927, respectively).

We subsequently fitted Model 5 in Mx using orthogonal target rotation. We expected the indicators of N (N1–N6) load on N, the indicators of E (E1–E6) load on E, and so on. A target matrix was specified containing ones in the positions consistent with our expectations, and zero elsewhere. We did not include any cross-loadings in the specification of the target matrix. The males served as the reference group. The factor loadings, factor (co)variances, and factor means are shown in Table 4. The chi-square test for the omnibus hypothesis that all latent mean difference are zero equals $\chi^2(5) = 69.5$, $p < .001$. In this solution, the latent mean differences with respect to N, E, A, and C are significant judging by the 99% confidence intervals and by the individual chi-square tests: N, $\chi^2(1) = 48.3$, $p < .01$; E, $\chi^2(1) = 9.4$,

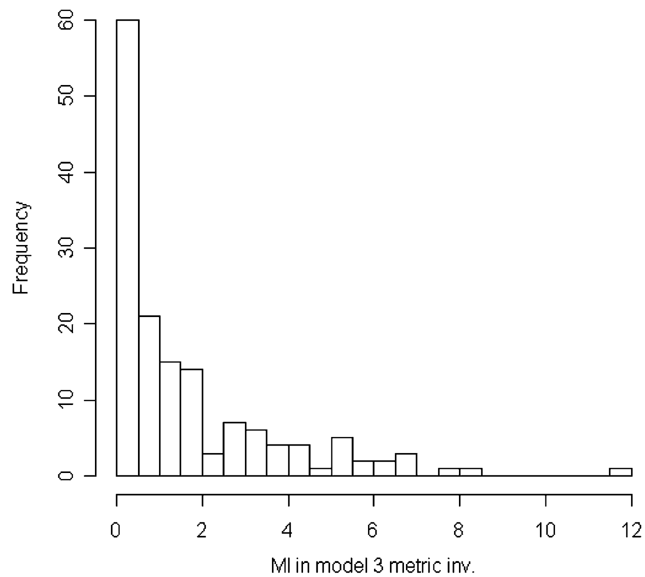

FIGURE 1 Modification indexes associated with the factor loading matrix (NEO-PI-R metric invariance model).

$p < .01$; O, $\chi^2(1) = 1.16$, *ns;* A, $\chi^2(1) = 16.6$, $p < .01$; C, $\chi^2(1) = 8.67$, $p < .01$. So given orthogonal target rotation, we find that in the Department of Psychology at the University of Amsterdam, female students score higher than male students with respect to N, E, A, and C.

Given that each common factor is supposed to have six definite indicators, there are 24 possible cross-loadings (e.g., the possible loadings of the six indicators of N on E, A, O, and C) per common factor. We find that the numbers of cross-loading are relatively large: 14 (N), 16 (E), 11 (E), 12 (A), and 13 (C). Although the one-at-a-time 99% confidence interval might be too liberal given the number of tests, the number of cross-loadings is such that a confirmatory approach is hardly feasible (McCrae et al., 1996). In addition the chi-square index of the confirmatory version of Model 5 equaled $\chi^2(833) = 3{,}342$ (RMSEA = .12, ECVI = 8.29, NNFI = .76, CFI = .77).[5] So regardless of the exact criterion used in establishing the significance of the individual cross-loadings, a purely confirmatory model is clearly rejected. Happily, the indicators (say, N1–N6), which are meant to load on given factors (say, N), do so with significant and relatively large factor loadings (N5 is an exception). This provides some basis for the identification of the common factors.[6]

[5]This is an oblique confirmatory oblique factor model in which all untargeted (cross-) factor loadings are fixed to zero, but with the same additional free parameters as in Model 5 in Table 3.

[6]An anomynous reviewer maintained that a confirmatory approach is possible, stating that "if NEO researchers would consistently try to replicate cross-loadings of the various NEO items, we would eventually get to the point where we would understand and a priori expect the large number of specific NEO items to cross-load." Clearly, the aim of this article is to present an exploratory, target rotated, multigroup factor model. Our illustration seems fitting in the light of McCrae et al. (1996), but we fully accept that a confirmatory approach might be possible at some point in the future.

TABLE 4
Factor Loadings Standardized in the Male Sample (Orthogonal Target Rotation)

	N	*E*	*O*	*A*	*C*	*SMC*
N1	***.810***	*−.212*	.093	.056	*−.162*	73.9
N2	***.519***	.022	−.002	*−.476*	*−.190*	53.4
N3	***.722***	*−.326*	.111	.059	*−.240*	70.1
N4	***.511***	*−.266*	−.083	.050	*−.136*	36.0
N5	***.347***	*.406*	*.144*	*−.106*	*−.383*	46.6
N6	***.719***	*−.184*	*−.128*	.053	*−.276*	64.7
E1	−.067	***.695***	*.168*	*.436*	.101	71.6
E2	−.025	***.598***	−.123	*.132*	*−.185*	42.6
E3	*−.228*	***.487***	.087	*−.353*	*.152*	44.5
E4	−.078	***.578***	*.181*	−.104	*.182*	41.7
E5	.022	***.413***	*.174*	*−.157*	*−.246*	28.7
E6	*−.171*	***.717***	*.210*	*.147*	.024	61.0
O1	*.161*	.102	***.491***	.034	*−.284*	36.0
O2	*.130*	*.153*	***.655***	*.142*	.068	49.5
O3	*.156*	*.334*	***.495***	*.186*	.079	42.2
O4	−.095	*.172*	***.416***	−.003	−.040	21.3
O5	*−.143*	−.105	***.624***	−.102	.096	44.1
O6	−.048	.025	***.411***	*.118*	−.041	18.7
A1	*−.237*	*.376*	*.135*	***.476***	.087	45.1
A2[a]	*−.118*	*−.179*	−.043	***.543***	*.191*	38.0
A3	−.085	*.293*	*.177*	***.582***	*.302*	55.6
A4	−.123	*−.287*	−.043	***.623***	.013	48.8
A5[a]	.048	*−.208*	−.065	***.423***	−.008	22.9
A6	*.125*	*.130*	*.277*	***.372***	−.029	24.9
C1	*−.364*	.094	.038	.001	***.615***	52.1
C2	−.107	−.011	*−.165*	.042	***.665***	48.3
C3	*−.257*	−.050	.015	*.279*	***.618***	52.9
C4	*−.169*	*.241*	.121	−.071	***.654***	53.5
C5	*−.379*	.103	.018	.066	***.702***	65.3
C6	*−.164*	*−.338*	*−.133*	*.176*	***.592***	54.1
Male var	1	1	1	1	1	
Female covar	1.609					
	−.220	.694				
	−.084	−.015	.878			
	−.179	−.026	.005	1.341		
	.497	−.033	.025	−.276	.988	
Male M	0	0	0	0	0	
Female M	.823	.323	−.124	.478	.327	
99% CI low:up	.48:1.19	.02:.63	−.46:.20	.14:.83	.01:.66	
$\chi^2(1)$	42.3	7.55	0.938(ns)	12.25	7.20	

Note. SMC = squared multiple correlation. The SMC is the percentage of variance in the indicator explained by the common factors. Bold factor loadings are the targeted factor loadings. Italicized factor loadings are statistically significant given the 99% one-at-a-time confidence intervals.

[a]Intercepts of facets A2 sincerity and A5 modesty were unconstrained over sex.

In conclusion, we find partial strong factorial invariance to be tenable in these data. On the basis of the LISREL results, we decided to relax the equality constraints of just 2 of the 30 intercepts (both indicators of Agreeableness, namely A2 sincerity and A5 modesty). On both of these facets, the females scored higher than would be accounted for by the factor mean difference. The mean differences on the remaining 28 facets can be interpreted in terms of the common factor mean differences.

It is important to emphasize that the exact results should be interpreted conditional on the choice of rotation criterion. To illustrate this, we repeated the analysis using oblique target rotation (using the same target and using the males as the reference group). We consider only the latent mean differences.[7] The point estimates of the latent mean differences were 0.634 (N), 0.149 (E), −.014 (O), .507 (A), and .111 (C). Judging by the one-at-a-time 99% confidence intervals, only the differences with respect to N and A are significant. The individual likelihood ratio tests were: N, $\chi^2(1) = 31.1$, $p < .01$; E, $\chi^2(1) = 1.99$, *ns*; O, $\chi^2(1) = 0.16$, *ns*; A, $\chi^2(1) = 17.8$, $p < .01$; C, $\chi^2(1) = 1.13$, *ns*. As expected, the test of the omnibus hypothesis of no latent differences was unaffected, $\chi^2(5) = 69.5$.

DISCUSSION

The aim of this article was to present an MG-EFA model to investigate measurement invariance in the multigroup exploratory factor model subject to target rotation. As demonstrated, measurement invariance can be tested readily using Mx[8] (Neale et al., 2003). The key to the method is that, once strong or strict factorial invariance has been established in the unrotated MG-EFA (Hessen et al., 2006), one can refit the model subject to the target rotation. The results thus obtained are interpretable and more informative. We emphasize that the issue of measurement invariance per se can be addressed without rotating the solution (Hessen et al., 2006).

In the light of McCrae et al. (1996), cross-cultural personality research would seem to be one fruitful area of application. To date, investigations of the factorial invariance of the NEO-PI-R, which measures the Big Five personality dimensions, have relied on congruence measures to demonstrate the equality of factor loadings. However, the use of congruence measures is inconvenient as they require a bootstrapping scheme for their evaluation (Chan et al., 1999; McCrae et al., 1996), and are not, to our knowledge, amenable to likelihood ratio testing. More important, the equality of factor loadings over groups is a necessary but not sufficient condition for measurement invariance. Measurement invariance implies constraints over groups on the factor loadings, intercepts, and residual variances (Meredith, 1993). The establishment of strong or strict factorial invariance is a necessary condition for the correct interpretation of mean differences in personality between nations in terms of the Big Five personality dimensions (McCrae & Costa, 1997).

We have limited this article to target rotations; that is, we have supposed that one does have a definite idea about which indicators should load on which common factors. The specification of target rotation poses no problem in Mx, as the rotation matrices can be obtained by closed form solutions (e.g., Schönemann, 1966). These solutions (i.e., Equations 12 and 13) can be specified

[7]Full results are available on request.

[8]Mx is freely available (see http://www.vcu.edu/mx).

easily in the Mx matrix language (see the Mx scripts in the Appendix). We are currently studying the feasibility of using Mx to fit factor models subject to other rotation criteria that do not admit a closed form solution (see Bernaards & Jennrich, 2005; Browne, 2001).

Principal component analysis (PCA) is often used as a form of EFA. However, it is important to note that our interest in measurement invariance precludes the use of PCA, because PCA does not constitute a measurement model in the sense that it relates observed scores to latent scores. Mellenbergh's definition of measurement invariance (or unbiasedness) is formulated explicitly in terms of the function, which links the observations to the hypothesized latent variables. PCA is therefore not suited to investigate measurement invariance. We do note that multigroup PCA, including structured means, has been developed by Flury, Nel, and Pienaar (1995; for a discussion in terms of the LISREL model, see Dolan, Bechger, & Molenaar, 1999).

The present use of Mx to carry out MG-EFA has several advantages, including the ability to handle missing data, using raw data likelihood estimation (Schafer & Graham, 2002), the ability to carry out power calculations in the rotated solution (Satorra & Saris, 1985), and the flexible extension to other models. Such models, which can also be considered in a single-group context, include longitudinal factor analysis and the analyses of sets of variables. The latter is akin to canonical correlation analysis, but employs the factor model as a measurement model, and target rotation to facilitate interpretation of the results. Finally, one might also consider estimation methods other than normal theory maximum likelihood. These possibilities have yet to be studied in detail.

REFERENCES

Basilevsky, A. (1983). *Applied matrix algebra in the statistical sciences.* Mineola, NY: Dover.

Bernaards, C. A., & Jennrich, R. I. (2005). Gradient projection algorithms and software for arbitrary rotation criteria in factor analysis. *Educational and Psychological Measurement, 65,* 676–696.

Bollen, K. A. (1989). *Structural equations with latent variables.* New York: Wiley.

Bollen, K. A., & Long, J. S. (1993). Introduction. In K. A. Bollen & J. S. Long (Eds.), *Testing structural equation models* (pp. 1–9). Newbury Park, CA: Sage.

Browne, M. W. (2001). An overview of analytic rotation in exploratory factor analysis. *Multivariate Behavioral Research, 36,* 111–150.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. In K. A. Bollen & J. S. Long (Eds.), *Testing structural equation models* (pp. 136–162). Newbury Park, CA: Sage.

Browne, M. W., Cudeck, R., Tateneni, K., & Mels, G. (1998). *CEFA: Comprehensive Exploratory Factor Analysis* [Document and computer software]. Retrieved April 5, 2004, from http://quantrm2.psy.ohio-state.edu/browne/software.htm

Chan, W., Ho, R. M., Leung, K., Chan, D. K., & Yung, Y. (1999). An alternative method for evaluating congruence coefficients with Procrustes rotation: A bootstrap procedure. *Psychological Methods, 4,* 378–402.

Chen, F. F., Sousa, K. H, & West, S. G. (2005). Testing measurement invariance of second-order factor models. *Structural Equation Modeling, 12,* 471–492.

Costa, P. T., & McCrae, R. R. (1992). *Revised NEO personality inventory (NEO-PI-R) and the NEO Five-Factor inventory (NEO-FFI) professional manual.* Odessa, FL: Psychological Assessment Resources.

Cudeck, R., & O'Dell, L. L. (1994). Application of standard error estimates in unrestricted factor analysis: Significance tests for factor loadings and correlations. *Psychological Bulletin, 115,* 475–487.

Dolan, C. V. (2000). A model-based approach to Spearman's hypothesis. *Multivariate Behavioral Research, 35,* 21–50.

Dolan, C. V., Bechger, T., & Molenaar, P. C. M. (1999). Using structural equation modeling to fit models incorporating principal components. *Structural Equation Modeling, 6,* 233–261.

Dolan, C. V., Roorda, W., & Wicherts, J. M. (2004). Two failures of Spearman's hypothesis: The GATB in Holland and the JAT in South Africa. *Intelligence, 32,* 155–173.

Ferrando, P. J., & Lorenzo-Seva, U. (2000). Unrestricted versus restricted factor analysis of multidimensional test items: Some aspects of the problem and some suggestions. *Psicologica, 21,* 301–323.
Flury, B. D., Nel, D. G., & Peinaar, I. (1995). Simultaneous detection of shift in means and variances. *Journal of the American Statistical Association, 90,* 1474–1481.
Hattie, J., & Fraser, C. (1988). The constraining of parameters in restricted factor analysis. *Applied Psychological Measurement, 12,* 115–162.
Hessen, D. J., Dolan, C. V, & Wicherts, J. M. (2006). Multi-group exploratory factor analysis and the power to detect uniform bias. *Applied Psychological Research, 30,* 233–246.
Horn, J. L., & McArdle, J. J. (1992). A practical and theoretical guide to measurement invariance in aging research. *Experimental Aging Research, 18,* 117–144.
Jöreskog, K. G. (1971). Simultaneous factor analysis in several populations. *Psychometrika, 36,* 409–426.
Jöreskog, K. G. (1978). Structural analysis of covariance and correlation matrices. *Psychometrika, 43,* 443–477.
Jöreskog, K. G. (1993). Testing structural equation models. In K. A. Bollen & J. S. Long (Eds.), *Testing structural equation models* (pp. 294–316). Newbury Park, CA: Sage.
Jöreskog, K. G., & Sörbom, D. (1999). *LISREL 8* [Computer program]. Chicago: Scientific Software International.
Lawley, D. N., & Maxwell, A. E. (1971). *Factor analysis as a statistical method.* London: Butterworth.
Little, T. D. (1997). Mean and covariance structures (MACS) analysis of cross-cultural data: Practical and theoretical issues. *Multivariate Behavioral Research, 32,* 53–76.
Little, T. D., Slegers, D. W., & Card, N. A. (2006). A non-arbitrary method of identifying and scaling latent variables in SEM and MACS models. *Structural Equation Modeling, 13,* 59–72.
Luria, A. R. (1978). *The working brain.* Middlesex, UK: Penguin.
McArdle, J. J., & Cattell, R. B. (1994). Structural equation models of factorial invariance in parallel proportional profiles and oblique confactor problems. *Multivariate Behavioral Research, 29,* 63–113.
McCrae, R. R., & Costa, P. T. (1997). Personality trait structure as a human universal. *American Psychologist, 52,* 509–516.
McCrae, R. R., Zonderman, A. B., Costa, P. T., Bond, M. H., & Paunonen, S. V. (1996). Evaluating replicability of factors in the Revised NEO Personality Inventory: Confirmatory factor analysis versus Procrustes rotation. *Journal of Personality and Social Psychology, 70,* 552–566.
McDonald, R. P. (1999). *Test theory: A unified treatment.* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
Mellenbergh, G. J. (1989). Item bias and item response. *International Journal of Educational Statistics, 13,* 127–143.
Meredith, W. (1993). Measurement invariance, factor analysis and factorial invariance. *Psychometrika, 58,* 525–543.
Millsap, R. E. (2001). When trivial constraints are not trivial: The choice of uniqueness constraints in confirmatory factor analysis. *Structural Equation Modeling, 8,* 1–17.
Millsap, R. E., & Everson, H. (1991). Confirmatory measurement model comparisons using latent means. *Multivariate Behavioral Research, 26,* 479–497.
Millsap, R. E., & Kwok, O. (2004). Evaluating the impact of partial factorial invariance on selection in two populations. *Psychological Methods, 9,* 93–115.
Neale, M. C., Boker, S. M., Xie, G., & Maes, H. H. (2003). *Mx: Statistical modeling* (6th ed.). Richmond, VA: Virginia Commonwealth University, Department of Psychiatry.
Neale, M. C., & Miller, M. B. (1997). The use of likelihood-based confidence intervals in genetic models. *Behavior Genetics, 27,* 113–120.
Satorra, A., & Saris, W. E. (1985). Power of the likelihood ratio test in covariance structure analysis. *Psychometrika, 50,* 83–90.
Schafer, J. L., & Graham, J. W. (2002). Missing data: Our view of the state of the art. *Psychological Methods, 7,* 147–177.
Schermelleh-Engel, K., Moosbrugger, H., & Müller, H. (2003). Evaluating the fit of structural equation models: Test of significance and descriptive goodness-of-fit measures. *Methods of Psychological Research, 8,* 23–74.
Schönemann, P. H. (1966). A generalized solution of the orthogonal procrustes problem. *Psychometrika, 31,* 1–10.
Sörbom, D. (1974). A general method for studying differences in factor means and factor structure between groups. *British Journal of Mathematical and Statistical Psychology, 27,* 229–239.
van der Sluis, S., Posthuma, D., Dolan, C. V., de Geus, E. J. C., Colom, R., & Boomsma, D. I. (2005). Sex differences on the Dutch WAIS-IIIR. *Intelligence, 34,* 273–289.
Wicherts, J. M., Dolan, C. V., Hessen, D. J., Oosterveld, P., van Baal, G. C. M., Boomsma, D. I., et al. (2004). Are intelligence tests measurement invariant over time? Investigating the nature of the Flynn effect. *Intelligence, 32,* 509–537.

APPENDIX
Mx SCRIPTS (ILLUSTRATION 1; SEE TABLES 1 AND 2)

Mx Script for Oblique Target Rotation

```
! start script
! oblique target rotation.
#ngroup 2
#define ny 6
#define ne 2
title strict fact. inv.
da no=250 ni=6
 cm fu
 1.140 0.642000 0.61600 0.35600 0.30400 0.210000
 0.642 1.163750 0.64425 0.44925 0.41025 0.309375
 0.616 0.644250 1.12800 0.46550 0.43300 0.333750
 0.356 0.449250 0.46550 1.12800 0.66050 0.577500
 0.304 0.410250 0.43300 0.66050 1.20600 0.626250
 0.210 0.309375 0.33375 0.57750 0.62625 1.062500
 me
 5.000 5.000000 5.00000 5.00000 5.00000 5.000000
!
begin matrices
L full ny ne fr       ! unrotated factor loadings
h diag ny ny fr       ! residual cov. matrix
t full ny ne fi       ! target
m full 1 ny fr        ! intercepts
a ful ne 1 fi         ! latent means
end matrices
!
begin algebra;
x=((l'*l)~*l'*t);     ! least squares solution
d=\sqrt( (\v2d( \d2v( (x'*x)~))));   ! scale
r=x*(d);                             ! rotation matrix - scaled
b=l*r;                               ! rotation
c=(r'*r)~;                           ! latent cor. matrix
end algebra;
cov B*C*B' + h*h /
me m + (B*a)' /
ma a
0 0
! the target
ma t
1 0
1 0
1 0
0 1
0 1
0 1
ma m
5 5 5 5 5 5
fi l 1 2
va 0 l 1 2
ma L
```

```
 0.75895 0.00000
 0.77476 0.14405
 0.74841 0.19206
 0.48488 0.67223
 0.43218 0.76826
 0.31623 0.72024
bo -10 10 l 1 1 - l ny ne
ma h
.6 .6 .6 .6 .6 .6
bo .01 10 h 1 1 - h ny ny
op it=5000
end
!
title strict fact. inv.
da no=250 ni=6
 cm fu
 1.076 0.588000 0.56800 0.36800 0.32800 0.240000
 0.588 1.121000 0.60750 0.47250 0.44550 0.348750
 0.568 0.607500 1.09700 0.49200 0.47100 0.375000
 0.368 0.472500 0.49200 1.18700 0.72600 0.637500
 0.328 0.445500 0.47100 0.72600 1.27700 0.690000
 0.240 0.348750 0.37500 0.63750 0.69000 1.118750
 me
 5.400 5.300000 5.25000 4.75000 4.65000 4.625000
!
begin matrices
B comp ny ne = B 1
r comp ne ne = r 1
h diag ny ny = h 1
y symm ne ne fr
a full ne 1 fr
m fu 1 ny = m 1
end matrices
begin algebra;
c=r~*y*r′~;
end algebra;
!
cov B*C*B′ + h*h /
me m + (B*a)′ /
ma a
.1 .1
ma y
1 .5 1
! interval @99 a 1 1 - a ne 1 b 1 1 - b ny ne
op issat mu
end
drop @0 a 2 1 1 a 2 2 1
op
end
! end script
```

Mx Script for Orthogonal Target Rotation

```
! start script
! orthogonal target rotation.
```

```
#ngroup 2
#define ny 6
#define ne 2
title MODEL 4 strict fact. inv. orthog target rotation
! group one is the references group.
da no=250 ni=6
cm fu
 1.140 0.642000 0.61600 0.35600 0.30400 0.210000
 0.642 1.163750 0.64425 0.44925 0.41025 0.309375
 0.616 0.644250 1.12800 0.46550 0.43300 0.333750
 0.356 0.449250 0.46550 1.12800 0.66050 0.577500
 0.304 0.410250 0.43300 0.66050 1.20600 0.626250
 0.210 0.309375 0.33375 0.57750 0.62625 1.062500
me
 5.000 5.000000 5.00000 5.00000 5.00000 5.000000
!
begin matrices
a full ne 1 fi         ! latent means = zero in ref group
l full ny ne fr        ! unrotated factor loadings (lambda)
h diag ny ny fr        ! diag. cov matrix of residual (theta)
t full ny ne fi        ! target rotation matrix
m full 1 ny fr         ! intercepts
end matrices
!
begin algebra;
x=(l′*t);
w=\evec(x*x′);         !eigenvalue decomp schonemann 1966 eq. 13
v=\evec(x′*x) ;        !eigenvalue decomp schonemann 1966 eq. 13
e=\d2v(w′*x*v);        !singular values
f=\abs(e);             !
g=\v2d(e)*(\v2d(f)~);  ! matrix to re-orient eigenvectors
r=(w*g)*v′;            ! rotation matrix
y=r′*r;                ! check: identity matrix
b=l*r;                 ! target rotated factor loadings
end algebra;
cov b*y*b′ + h*h′ /    ! covariance matrix
me m + (B*a)′ /        ! mean vector
ma a
0 0
! the target matrix
ma t
1 0
1 0
1 0
0 1
0 1
0 1
ma m
5 5 5 5 5 5
fi l 1 2
va 0 l 1 2
ma L
 0.5 0.0
 0.5 0.1
```

```
 0.5 0.1
 0.4 0.3
 0.4 0.4
 0.3 0.5
ma h
.6 .6 .6 .6 .6 .6
bo -10 10 l 1 1 - l ny ne
bo -5 5 h 1 1 - h ny ny
op it=5000
end
!
title MODEL 4 strict fact. inv.
da no=250 ni=6
cm fu
 1.076 0.588000 0.56800 0.36800 0.32800 0.240000
 0.588 1.121000 0.60750 0.47250 0.44550 0.348750
 0.568 0.607500 1.09700 0.49200 0.47100 0.375000
 0.368 0.472500 0.49200 1.18700 0.72600 0.637500
 0.328 0.445500 0.47100 0.72600 1.27700 0.690000
 0.240 0.348750 0.37500 0.63750 0.69000 1.118750
me
 5.400 5.300000 5.25000 4.75000 4.65000 4.625000
!
begin matrices
B comp ny ne = B 1
r comp ne ne = r 1
h diag ny ny = h 1
y symm ne ne fr
a full ne 1 fr
m fu 1 ny = m 1
end matrices
begin algebra;
c=r′*y*r ;
end algebra
!
cov B*c*B′ +h*h′ /
me m + (B*a)′ /
ma a
.1 .1
ma y
1 .0 1
! interval @99 a 1 1 - a ne 1 b 1 1 - b ny ne
op issat mu
end
drop @0 a 2 1 1 a 2 2 1
op
end
! end script
```